REVISED SIMPLEX METHOD

Simplex Tableau

ω	$c_B \bar{b}$
B^{-1}	$\bar{b}$

Example

Minimize $\quad -x_1 - 2x_2 + x_3 - x_4 - 4x_5 + 2x_6$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 6$$
$$2x_1 - x_2 - 2x_3 + x_4 \leq 4$$
$$x_3 + x_4 + 2x_5 + x_6 \leq 4$$

Introduce x_7, x_8 and x_9

Intial basis, $B = [\vec{a}_7, \vec{a}_8, \vec{a}_9] = \begin{array}{c} \begin{array}{ccc} a_7 & a_8 & a_9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} = I_3$

Also, $\omega = C_B B^{-1} = (0,0,0)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (0,0,0)$

and, $\bar{b} = b = \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$, $\quad C_B \bar{b} = (0,0,0)\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix} = 0$

Iteration 1

Basis Inverse

	0	0	0	0
x_7	1	0	0	6
x_8	0	1	0	4
x_9	0	0	1	4

For nonbasic variables, find $z_j - c_j = \omega a_j - c_j$

$$z_1 - c_1 = \omega a_1 - c_1 = (0,0,0)\begin{bmatrix}1\\2\\0\end{bmatrix} - (-1) = 1$$

$$z_2 - c_2 = \omega a_2 - c_2 = (0,0,0)\begin{bmatrix}1\\-1\\0\end{bmatrix} - (-2) = 2$$

$$z_3 - c_3 = \omega a_3 - c_3 = (0,0,0)\begin{bmatrix}1\\-2\\1\end{bmatrix} - (1) = -1$$

$$z_4 - c_4 = \omega a_4 - c_4 = (0,0,0)\begin{bmatrix}1\\1\\1\end{bmatrix} - (-1) = 1$$

$$z_5 - c_5 = \omega a_5 - c_5 = (0,0,0)\begin{bmatrix}1\\0\\2\end{bmatrix} - (-4) = 4$$

$$z_6 - c_6 = \omega a_6 - c_6 = (0,0,0)\begin{bmatrix}1\\0\\1\end{bmatrix} - (2) = -2$$

$\text{Max}\ (z_j - c_j) = 4 \Rightarrow$ Variable is x_5

Calculate $y_5 = B^{-1} a_K = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}1\\0\\2\end{bmatrix}$

Insert the vector

$$\begin{bmatrix} z_5 - c_5 \\ \hline y_5 \end{bmatrix} = \begin{bmatrix} 4 \\ \hdashline 1 \\ 0 \\ 2 \end{bmatrix}$$

to the right of tableau and pivot at $y_{35} = 2$ (Min Ratio)

Basis Inverse			RHS	x_5
0	0	0	0	4
1	0	0	6	1
0	1	0	4	0
0	0	1	4	2

Pivoting means an identity column for entering variable column.

$$\begin{array}{ccc} \text{Before Pivoting} & & \text{Aftering Pivoting} \\ x_5 & & x_5 \\ \begin{bmatrix} 4 \\ \hline 1 \\ 0 \\ 2 \end{bmatrix} & \rightarrow & \begin{bmatrix} 0 \\ \hline 0 \\ 0 \\ 1 \end{bmatrix} \end{array}$$

Basis Inverse

Old Table

				RHS	
row-0	0	0	0	0	4
row-1	1	0	0	6	1
row 2	0	1	0	4	0
row 3	0	0	1	4	2

Divide all entries of in last row by 2

New Table

0	1	0	4	0
0	0	$\frac{1}{2}$	2	1

Divide all entries of row 2 by -2 and add to row 1.

1	0	$-\frac{1}{2}$	4	0
0	1	0	4	0
0	0	$\frac{1}{2}$	2	1

Multiply all entries of row 2 by -2 and add to row 0 entries

	0	0	-2	-8	0
x_7	1	0	$-\frac{1}{2}$	4	0
x_8	0	1	0	4	0
x_5	0	0	$\frac{1}{2}$	2	1

$$\omega = c_B B^{-1} = \begin{array}{ccc} c_7 & c_8 & c_5 \\ (0 & 0 & -4) \end{array} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$= (0 \quad 0 \quad -2)$$

$$z_0 = c_B B^{-1} b$$

$$= \omega b = (0 \quad 0 \quad -2) \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix} = -8$$

$$\bar{b} = B^{-1} b$$

$$= \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$$

So, all updated entries are automatically present in the table

$\vec{\omega}$	$c_B \bar{b}$
B^{-1}	$\bar{b}$

After pivoting

	Basis Inverse			RHS
z	0	0	-2	-8
x_7	1	0	$-\frac{1}{2}$	4
x_8	0	1	0	4
x_5	0	0	$\frac{1}{2}$	2

x_5
0
0
0
1

Iteration 2:

For each nonbasic variable, calculate $z_j - c_j$

$$z_1 - c_1 = \omega a_1 - c_1 = (0,0,-2)\begin{bmatrix}1\\2\\0\end{bmatrix} - (-1) = 1$$

$$z_2 - c_2 = \omega a_2 - c_2 = (0,0,-2)\begin{bmatrix}1\\-1\\0\end{bmatrix} - (-2) = 2$$

$$z_3 - c_3 = \omega a_3 - c_3 = (0,0,-2)\begin{bmatrix}1\\-2\\1\end{bmatrix} - (1) = -3$$

$$z_4 - c_4 = \omega a_4 - c_4 = (0,0,-2)\begin{bmatrix}1\\1\\1\end{bmatrix} - (-1) = -1$$

$$z_6 - c_6 = \omega a_6 - c_6 = (0,0,-2)\begin{bmatrix}1\\0\\1\end{bmatrix} - (2) = -4$$

$$z_9 - c_9 = \omega a_9 - c_9 = (0,0,-2)\begin{bmatrix}0\\0\\1\end{bmatrix} - 0 = -2$$

Max $(z_j - c_j) = 2 \Rightarrow$ variable is x_2

Calculate $y_2 = B^{-1} a_k = \begin{bmatrix}1 & 0 & -\frac{1}{2}\\0 & 1 & 0\\0 & 0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$

Insert vector

$$\begin{bmatrix}\dfrac{z_2 - c_2}{y_2}\end{bmatrix} = \begin{bmatrix}2\\ \hline 1\\-1\\0\end{bmatrix}$$

to the right of Tableau and pivot at y_{12}

	Basis Inverse			RHS		x_2	
z	0	0	-2	-8		2	
$\leftarrow x_7$	1	0	$-\frac{1}{2}$	4		1	y_{12}
x_8	0	1	0	4		-1	
x_5	0	0	$\frac{1}{2}$	2		0	

After Pivoting

	Basis Inverse			RHS		x_2
z	-2	0	-1	-16		0
x_2	1	0	$-1/2$	4		1
x_8	1	1	$-1/2$	8		0
x_5	0	0	$1/2$	2		0

Iteration 3

$$w = (-2, 0, -1)$$

$$z_1 - c_1 = wa_j - c_j = (-2, 0, -1)\begin{bmatrix}1\\2\\0\end{bmatrix} - (-1) = -1$$

$$z_3 - c_3 = wa_3 - c_3 = (-2, 0, -1)\begin{bmatrix}1\\-2\\1\end{bmatrix} - 1 = -4$$

$$z_4 - c_4 = w a_4 - c_4 = (-2, 0, -1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - (-1) = -2$$

$$z_6 - c_6 = w a_6 - c_6 = (-2, 0, -1)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 2 = -5$$

$$z_9 - c_9 = (-2, 0, -1)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 0 = -1$$

Max $(z_j - c_j) = -1$ which is NOT +ve

Basic feasible solution is OPTIMAL.